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EE 513 – Modern Control Theory

Winter 2026

STATE-SPACE MODELING AND CONTROL OF SATELLITE ATTITUDE SYSTEM

MOTIVATION

Why Attitude Control Matters

- Satellites must point precisely:
  - Antennas
  - Cameras
  - Solar Panels
- Reaction wheels generate internal torque
- Thrusters provide external torque authority
- Modern control enables guaranteed stability

PHYSICAL MODEL

System Components:

- Spacecraft Inertia J_s $\left(\frac{\mathrm{kg}}{\mathrm{m}^2}\right)$
- Reaction wheel inertia J_w $\left(\frac{\mathrm{kg}}{\mathrm{m}^2}\right)$
- Wheel torque τ_w (Nm)
- Thruster torque τ_t (Nm)

Dynamics:

$$J_s\ddot{\theta} = -\tau_w + \tau_t$$

$$J_w\dot{\omega} = \tau_w$$

States:

$$x_1 = \theta$$

$$x_2 = \dot{\theta}$$

$$x_3 = \omega_w$$

State-Space Model

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ -\frac{1}{J_s} & \frac{1}{J_s} \\ \frac{1}{J_w} & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D = 0$$

$$u = \begin{bmatrix} \tau_w \\ \tau_t \end{bmatrix}$$

$$y = \begin{bmatrix} \theta s \\ \omega_w \end{bmatrix}$$

Transfer Function

$$G(s) = \frac{Y(s)}{U(s)} = \begin{bmatrix} \frac{\theta(s)}{T_w(s)} & \frac{\theta(s)}{T_t(s)} \\ \frac{\Omega_w(s)}{T_w(s)} & \frac{\Omega_w(s)}{T_t(s)} \end{bmatrix} = \begin{bmatrix} -\frac{1}{J_s s^2} & \frac{1}{J_s s^2} \\ \frac{1}{J_w s} & 0 \end{bmatrix}$$

Key Observations

- $\frac{\theta(s)}{\tau(s)} = -\frac{1}{J_s s^2}$ => Spacecraft angle is a double integrator
- Poles at s = 0 (multiplicity 2) => Marginally unstable
- $\frac{\omega_s(s)}{\tau_w(s)} = \frac{1}{J_w s}$ => Reaction wheel is a single integrator
- External torque τ_t directly drives spacecraft angle

Open-Loop Stability

Eigenvalues of A:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\lambda = 0, 0, 0}$$

m = 3

q = 2, k = 2

Interpretation:

- 2 Jordan blocks, 2x2 and 1x1
- Double integrator, plus separate constant state
- Unstable, linear drift without control:

$$x_1(t) = x_1(0) + x_2(0)t$$

CONTROLLABILITY ANALYSIS

Controllability matrix:

$$P = [B \quad AB \quad A^2B]$$

Compute:

$$AB = \begin{bmatrix} -\frac{1}{J_s} & \frac{1}{J_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2B = 0$$

$$a = \frac{1}{J_s}, \qquad b = \frac{1}{J_w}$$

$$P = \begin{bmatrix} 0 & 0 & -a & a & 0 & 0 \\ -a & a & 0 & 0 & 0 & 0 \\ b & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank(c) = 3

STATE FEEDBACK CONTROL

Closed-loop system:

$$\dot{x} = (A - BK)x$$

Choose desired poles:

$$p = -0.4, -0.6, -1.5$$

Result:

- Stable, Overdamped, All real poles
- Slower, No Oscillation
- Beneficial for Antenna and High-Resolution Imaging
- Overshoot ≈ 0%
- Settling Time ≈ 7 s
- Rise Time ≈ 4.5 s
- Controlled wheel momentum

Controller:

$$u = -Kx + Fr$$

$r = \theta_{ref}$ (step input)

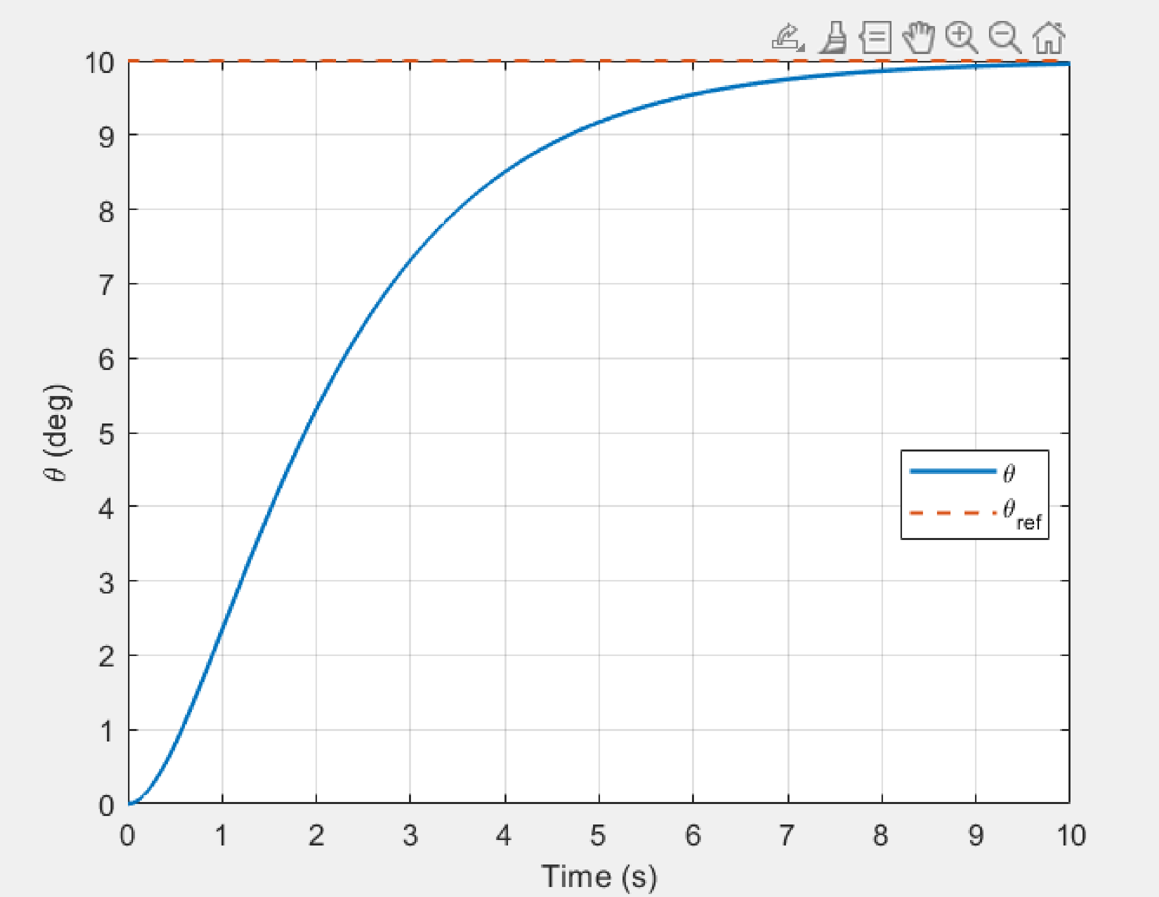
State Feedback Gain Matrices (MATLAB place(A, B, p)):

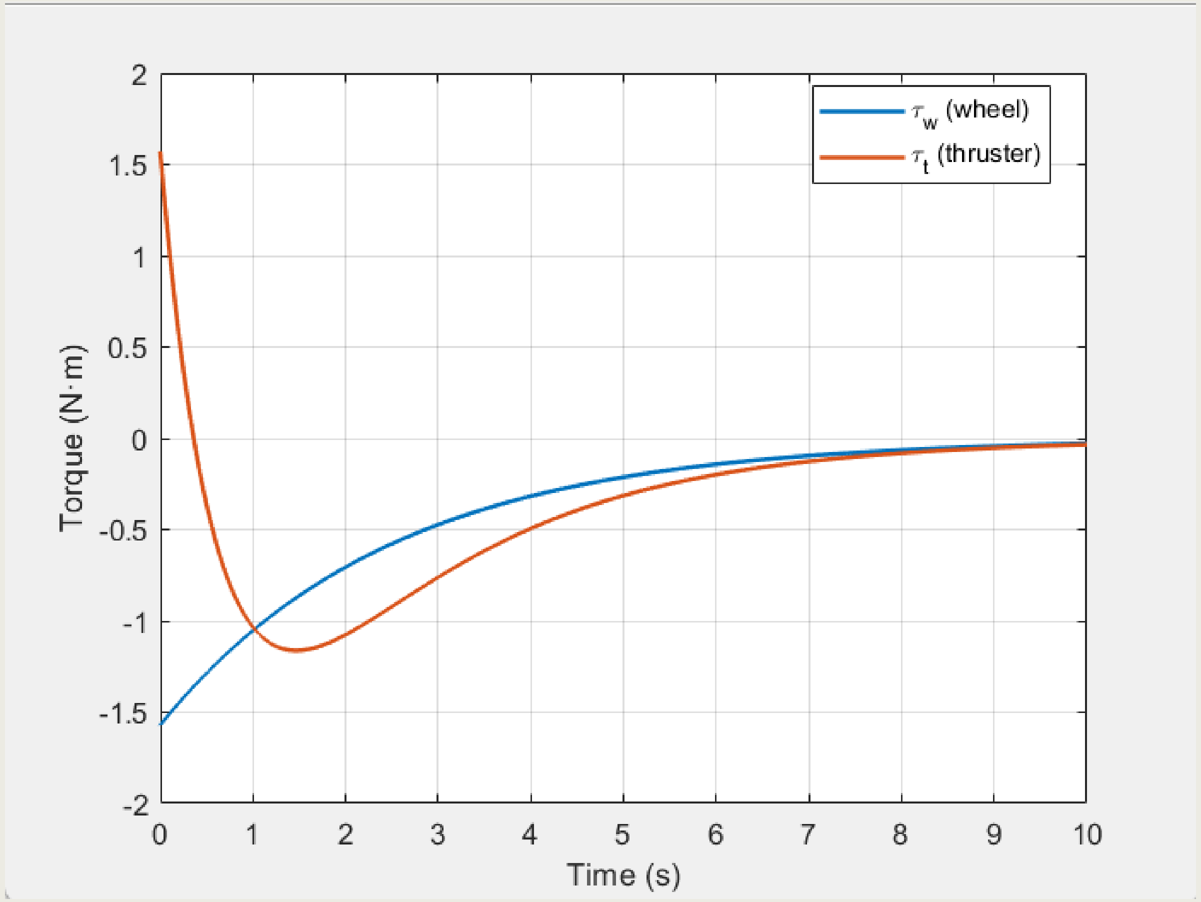
$$K = \begin{bmatrix} 0 & 0 & 0.04 \\ 18 & 42 & 0.04 \end{bmatrix}$$

Reference Gain Matrix:

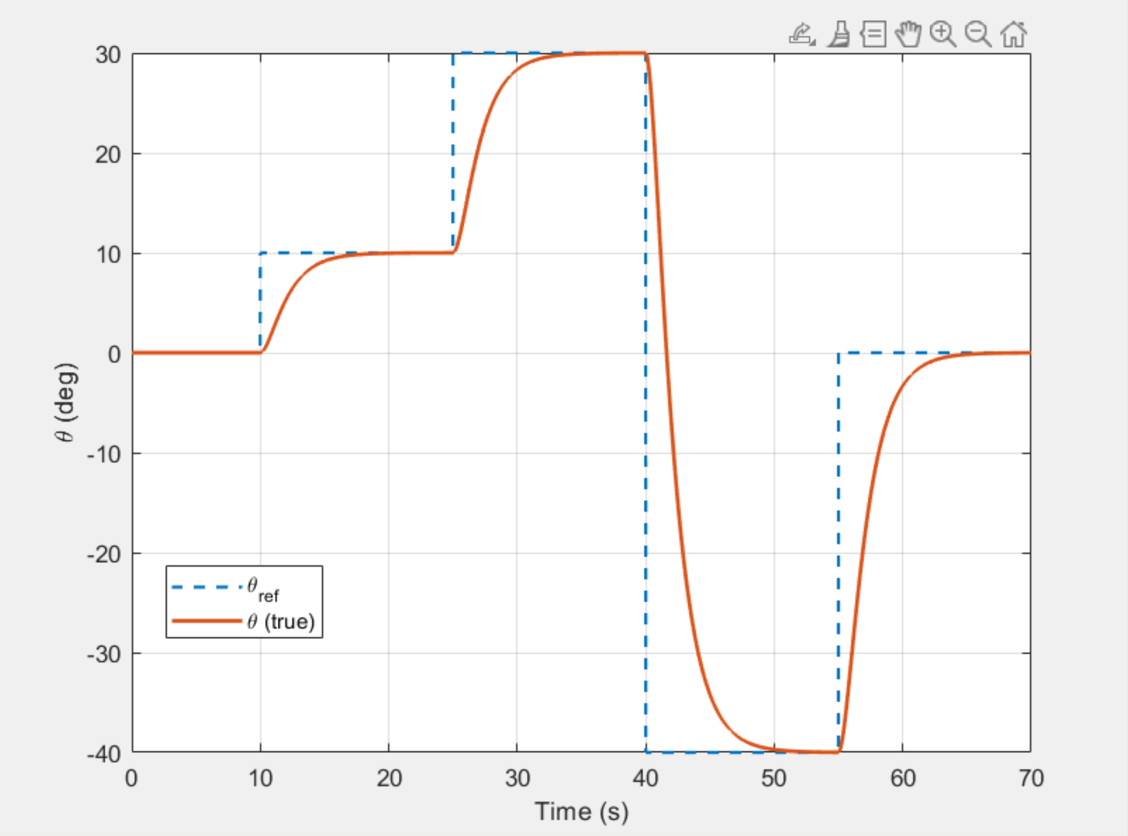
$$F = \begin{bmatrix} -9 \\ 9 \end{bmatrix}$$

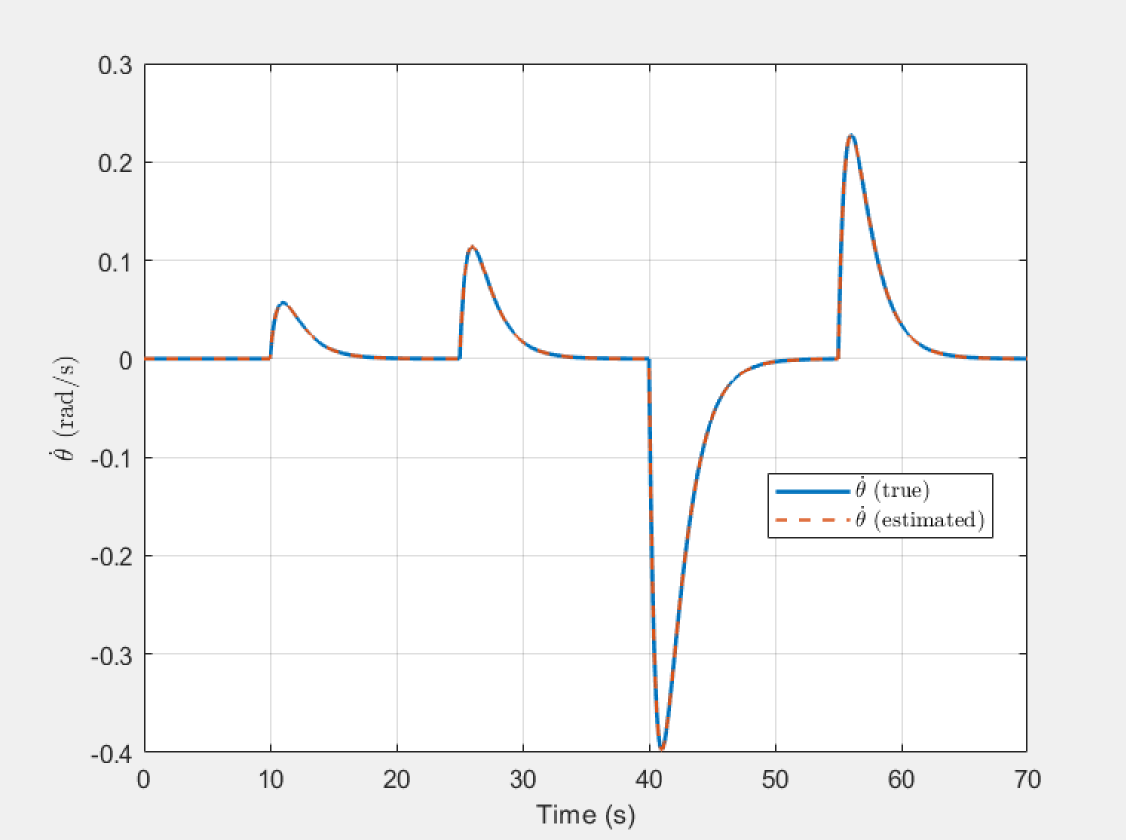
θref = 10 deg
Js = 20 kgm^2
Jw = 0.1 kgm^2





θref = 0; +10; +30; -40; 0 deg
Js = 20 kg/m^2
Jw = 0.1 kg/m^2





OBSERVABILITY ANALYSIS

Measured outputs:

$$y = \begin{bmatrix} \theta \\ \omega_w \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Observability matrix:

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

Compute:

$$O = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank(O) = 3

OBSERVER DESIGN

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Measured:

- θ
- ω_w

Not Measured:

- $\dot{\theta}$

For observer poles at -1, -1.5, -3:

$$L = \begin{bmatrix} 4.5 & 0 \\ 4.5 & 0 \\ 0 & 1 \end{bmatrix}$$

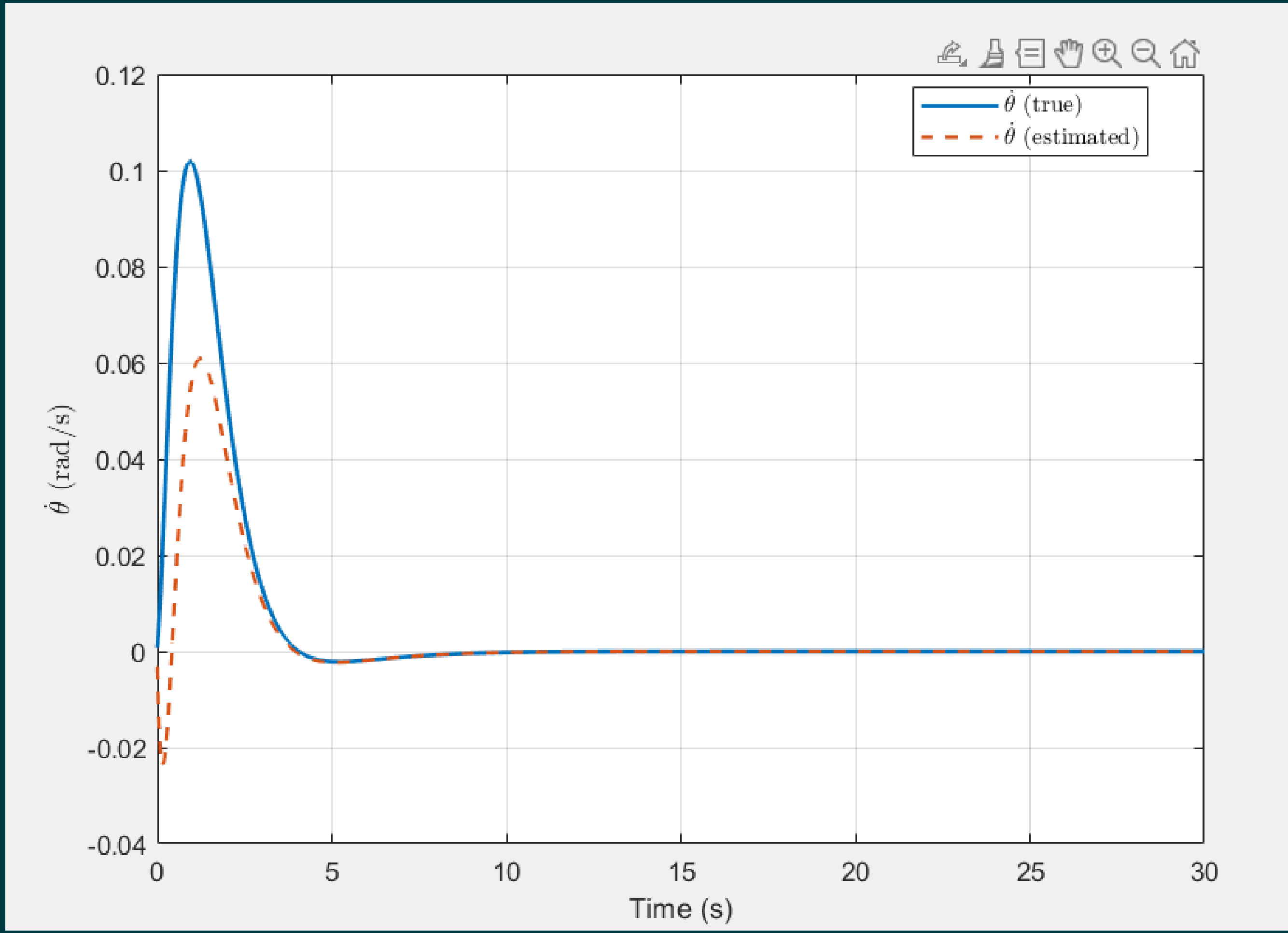
$$\dot{\hat{\theta}} = \hat{\dot{\theta}} + L_1(\theta - \hat{\theta})$$

$$\dot{\hat{\dot{\theta}}} = -\frac{1}{J_s}\tau_m + \frac{1}{J_s}\tau_t + L_2(\theta - \hat{\theta})$$

$$\dot{\widehat{\omega_w}} = \frac{1}{J_w}\tau_m + L_3(\omega_w - \widehat{\omega_w})$$

$$\dot{e} = (A - LC)e$$

$\theta ref = 10$ deg
$J_s = 20$ kgm^2
$J_w = 0.1$ kgm^2



THANK YOU